

Atomic–Molecular Condensates with Large Positive Scattering Length

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We show that in the limit of large and positive atom–atom scattering length the properties of an atomic–molecular Bose–Einstein Condensate (amBEC) are determined by an universal energy density functional. At temperatures lower than the binding energy of the shallow molecules (dimers) the chemical potentials for the atoms and molecules can be specified independently. Besides three–body recombinations processes, inelastic processes involving the formation of deep bound molecular states are possible.

In order to derive the energy density of an atom–molecule gas let us first consider the dilute regime where $nr_0^3 \ll 1$, with n a typical atomic or molecular density. The particle momenta relevant in this regime are of the order of $p = \hbar k \approx \hbar/n^{1/3} \ll \hbar/r_0$ thus the details of the potential are not important and the atom interactions can be described by a contact term

$$H_a = -\psi_a^\dagger \frac{\hbar^2 \nabla^2}{2m} \psi_a + \frac{1}{2} \lambda_2 \psi_a^\dagger{}^2 \psi_a^2 + \frac{1}{3} \lambda_3 \psi_a^\dagger{}^3 \psi_a^3,$$

where ψ_a^\dagger, ψ_a are creation and annihilation operators for atoms. The value of λ_2 is determined by the scattering length a . λ_3 depends also on a genuinely three-body length scale, denoted here by a_3 . The hamiltonian in Eq. (1) contains, in principle, all the information necessary to describe the system in the dilute regime, including the molecular states. In the case $a \gg r_0$ we are interested in, the presence of bound states within the regime of validity of Eq. (1) evidences the fact that perturbation theory breaks down. Thus, it is inconvenient to use it directly.

We can now consider now a regime even more dilute, where $na^3 \ll 1$. The relevant typical momenta are of the order $p = \hbar k \approx \hbar/n^{1/3} \ll \hbar/a \ll \hbar/r_0$. We can argue

though that transitions altering the number of molecular states are “slow” in this regime and, consequently, for time scales shorter than the transition rate, the number of atoms and molecules are separately conserved. A hamiltonian describing the system in this regime is

$$H_{am} = -\psi_a^\dagger \frac{\hbar^2 \nabla^2}{2m} \psi_a - \psi_m^\dagger \left(\frac{\hbar^2 \nabla_a^2}{4m} - \epsilon \right) \psi_m + \frac{1}{2} \lambda_{aa} \psi_a^\dagger{}^2 \psi_a^2 + \lambda_{am} \psi_a^\dagger \psi_m^\dagger \psi_a \psi_m + \frac{1}{2} \lambda_{mm} \psi_m^\dagger{}^2 \psi_m^2$$

Both H_a and H_{am} are applicable in the $na^3 \ll 1$ ($p \ll \hbar/a$) regime. Despite its smaller validity range ($na^3 \ll 1$ versus $nr_0^3 \ll 1$), H_{am} is useful since we can do perturbation theory with it since all non-perturbative physics occurring at the scale $\sim a$ leading to the formation of the bound state and the Efimov effect is encapsulated in the constants $\lambda_{aa}, \lambda_{am}$ and λ_{mm} . This leads to the determination of the constants of Eq. (1) in terms of those in Eq. (1), and we have provided the specific non-perturbative formulae valid in the regime of interest.

Particularly interesting are however the inelastic atom–dimer and dimer–dimer collisions. If the temperature of the system is significantly lower than the dimer binding energy, these inelastic collisions are not going to lead to a heating of the system, but mostly to its depletion. If an atom and a dimer collide and the dimer forms a deeper bound state, a large amount of energy is released and that the shallow dimer binding energy is significantly larger than the interaction energy as well. The momentum of the outgoing atom and of the deeply bound dimer are of the order $\hbar/r_0 \gg \hbar/a$ and such fast atoms and dimers will interact weakly with the rest of the atoms and dimers.